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SOLUTION BY B. F. YANNEY, College of Wooster, Ohio.

Suppose that the theorem is not true, and that  $n$  is the greatest number of consecutive terms in the progression no one of which is a prime.

Consider any  $n + 1$  consecutive terms of the progression, as

$$A_1 = a_1 + kd, A_2 = a_1 + (k + 1)d, A_3 = a_1 + (k + 2)d, \dots, A_{n+1} = a_1 + (k + n)d.$$

Set  $M = A_1 A_2 A_3 \dots A_{n+1}$ . Then will  $A_1' = A_1(M + 1)$ ,  $A_2' = A_1(M + 1) + d$ ,  $\dots$ ,  $A_{n+1}' = A_1(M + 1) + nd$  be  $n + 1$  consecutive terms of the progression, no one of which is a prime. For consider any one of them, as

$$A_{r+1}' = A_1(M + 1) + rd = (a_1 + kd)(M + 1) + rd = (a_1 + kd + rd)(M + 1) - rdM,$$

which is evidently not prime, since  $M$  is a multiple of  $a_1 + kd + rd$ . We are thus led to a contradiction. Hence the denial of the theorem must be withdrawn, and the theorem is true.

Also solved by H. N. CARLETON, ELIJAH SWIFT, HORACE OLSON and LOUIS CLARK.

**269 (Number Theory). Proposed by ARTEMAS MARTIN, Washington, D. C.**

Find three rectangular parallelepipeds whose edges are rational whole numbers, and whose solid diagonals are equal, and rational whole numbers.

I. SOLUTION BY THE PROPOSER.

Let  $w$ ,  $x$  and  $y$  denote the lengths of the edges, and  $z$  the solid diagonal, of any one of the three required solids; then we must have

$$z^2 = w^2 + x^2 + y^2.$$

Put  $x = np$ ,  $y = nq$ ,  $z = w + nr$ ; then

$$(w + nr)^2 = w^2 + (np)^2 + (nq)^2 = w^2 + 2nrw + n^2r^2,$$

which gives, after dividing by  $n$ ,

$$w = \frac{n(p^2 + q^2 - r^2)}{2r}, \quad \text{and} \quad z = \frac{n(p^2 + q^2 + r^2)}{2r}.$$

Now take  $n = 2r$  and we get the integral values

$$z = p^2 + q^2 + r^2, \quad w = p^2 + q^2 - r^2, \quad x = 2pr, \quad y = 2qr,$$

for one of the solids. The other two solids are obtained by interchanging the values of  $p$ ,  $q$ ,  $r$  in the expressions for  $w$ ,  $x$ , and  $y$ .

Hence

$$\begin{aligned} (p^2 + q^2 + r^2)^2 &= (p^2 + q^2 - r^2)^2 + (2pr)^2 + (2qr)^2 \\ &= (p^2 + r^2 - q^2)^2 + (2pq)^2 + (2qr)^2 = (q^2 + r^2 - p^2)^2 + (2pq)^2 + (2pr)^2. \end{aligned}$$

Take  $p = 4$ ,  $q = 2$ ,  $r = 1$ ; then the solids are

$$21, 19, 8, 4; 21, 16, 13, 4; 21, 16, 11, 8 \text{ (diagonals 21).}$$

Take  $p = 4$ ,  $q = 3$ ,  $r = 2$ ; then they are

$$29, 21, 16, 12; 29, 24, 12, 11; 29, 24, 16, 3 \text{ (diagonals 29).}$$

The values of  $p$ ,  $q$ ,  $r$  may be chosen at pleasure.

II. SOLUTION BY C. F. GUMMER, Queen's University, Kingston.

We have to find three solutions of the Diophantine equation,

$$r^2 - z^2 = x^2 + y^2, \tag{1}$$

having the value of  $r$  in common.

From the theory of the form  $x^2 + y^2$ , the general solution of (1) is given by

$$\begin{aligned} r + z &= m(a^2 + b^2), & r - z &= m(c^2 + d^2), \\ x &= m(ac + bd), & y &= m(ad - bc), \end{aligned}$$

where at most one of the integers  $a, b, c, d$  is zero, and either  $m$  or  $a^2 + b^2 + c^2 + d^2$  is even. It follows that

$$r = \frac{1}{2}m(a^2 + b^2 + c^2 + d^2), \quad z = \frac{1}{2}m(a^2 + b^2 - c^2 - d^2).$$

By permutation of  $a, b, c, d$  we obtain six solutions (generally distinct) with  $r$  in common from which three may be selected with no other value in common.

Thus, if  $m = 2, a = 5, b = 3, c = 1, d = 2$ , we get for  $x, y, z$  the sets of values

$$22, 14, 29; 34, 2, 19; 26, 26, 13;$$

$$26, 2, 29; 22, 26, 19; 34, 14, 13;$$

of which the first or last three are entirely distinct, and in each case the sum of the squares is  $39^2$ .

Evidently, by taking for  $r$  a value having various divisors expressed in the form  $a^2 + b^2 + c^2 + d^2$ , we can obtain any desired number of rectangular parallelepipeds with integral sides and a common integral diagonal.

Also solved by ELIJAH SWIFT, B. F. YANNEY, H. C. FEEMSTER and J. L. RILEY.

## QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

### DISCUSSIONS.

#### I. RELATING TO THE DERIVATION OF THE DISTANCE FORMULA.

By H. T. BURGESS, University of Wisconsin.

The derivation of the distance formula independent of Hesse's "normal form" as presented by Mr. Mathews on page 476 of the December MONTHLY appeals to me as an excellent idea. Hesse's form in this connection may well be relegated to Professor Miller's collection of "Obsoletes."<sup>1</sup>

Since the form  $y = mx + b$  includes all lines not parallel to the  $Y$ -axis and is the only one ever needed in ordinary straight-line problems, I suggest the derivation below as a possible alternative.

Given a point  $P = (x_1, y_1)$  and a line  $y = mx + b$ , to find the distance  $d$  from the point to the line.

Write the equation of the parallel through  $P$  in the form  $y = mx + b'$ . When a drawing is made, it is obvious that, numerically,

$$d = (b' - b) \cos \alpha = \frac{b' - b}{\sqrt{1 + m^2}},$$

where  $\alpha = \arctan m$ .

If one must be conventional, we have

$$d = \frac{b' - (\pm b)}{\pm \sqrt{1 + m^2}} = \frac{y_1 - mx_1 - (\pm b)}{\pm \sqrt{1 + m^2}}$$

to take care of the sign of  $d$ .

<sup>1</sup> Cf. pp. 453-456 in the December, 1917, MONTHLY.